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# A unified semiclassical theory of parallel and perpendicular giant magnetoresistance in metallic superlattices

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**Abstract.** The giant magnetoresistance effect in magnetic superlattices for the current perpendicular to and in the layer planes is studied within a unified semiclassical approach that is based on the Boltzmann equation with exact boundary conditions for the spin-dependent distribution functions of electrons. Interface processes responsible for the magnetoresistance are found to be different in these geometries, and this can result in an essential difference in general behaviour between the in-plane magnetoresistance and the perpendicular-plane one. A correlation between the giant magnetoresistance and the multilayer magnetization is also discussed.

## 1. Introduction

There has been great interest recently in magnetic superlattices displaying a wide array of fascinating properties. The most attractive phenomenon is the giant magnetoresistance (GMR) effect in which there is a drastic decrease in the multilayer resistance when the magnetic ordering of the superlattice is changed by the application of an external magnetic field [1]. In the majority of experiments the current flows in a direction parallel to the layer planes and by now an extensive literature has evolved that traces the GMR for the CIP (current-in-plane) geometry (see, for example, review [2] and references therein). In contrast, few experiments have been performed with the current flowing perpendicular to the layer planes (CPP geometry) [3–5]. When experimental results for the CIP geometry are compared with those for the CPP geometry, it is apparent that, although both the CIP-GMR and the CPP-GMR result from spin-dependent scattering, the former differs essentially from the latter in general behaviour (GMR magnitude, magnetic field dependence, temperature dependence). To gain some insight into what governs this difference, they should both be considered within a unified theory.

Whereas the theoretical understanding of the in-plane GMR has been greatly advanced and both semiclassical [6] and quantum [7, 8] models have been worked out in detail, few papers have been concerned with the CPP-GMR theory [9–12]. The main difficulty for a theoretical description arises from the necessity to take into account a spin accumulation effect and an electric field inhomogeneity that are characteristic of the CPP geometry.

The problem has been considered in some detail by Valet and Fert [12]. Their model is based on the Boltzmann kinetic equation with spin-dependent (no spin-flip) and spin-flip bulk scattering and accounts for interface scattering through phenomenological macroscopic parameters designating spin-dependent interface scattering. Unfortunately, their account of interface scattering is too crude to be used in a derivation of the unified theory.

In this paper we give a comparative description of the GMR in both geometries on the basis of a unified microscopic model within a semiclassical approach and analyse the

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correlation between the magnetoresistance $\rho$ and the superlattice magnetization $M$. As opposed to [12], the interaction of conduction electrons with the interfaces is described through exact boundary conditions for the spin-dependent distribution function of electrons.

## 2. Basic equations

Let us consider an infinite superlattice consisting of single-domain ferromagnetic layers with magnetic moments $M_i$ in the layer space, each layer being $L$ in thickness. The non-magnetic-interlayer thickness is assumed negligible compared with $L$. Neighbouring magnetic moments $M_i$ and $M_{i+1}$ will be considered antiparallel in the initial state (figure 1($a$)). An external magnetic field $H$ applied in the layer plane rotates the magnetic moments to the parallel arrangement and changes the angle $\theta = \theta(H)$ between $M_i$ and $M_{i+1}$ (figures 1($b$) and ($c$)). The relative superlattice magnetization $\mu$ is given by

$$\mu(H) = M(H)/M_s = \cos(\theta/2) \tag{1}$$

where $M_s$ is the saturation magnetization.

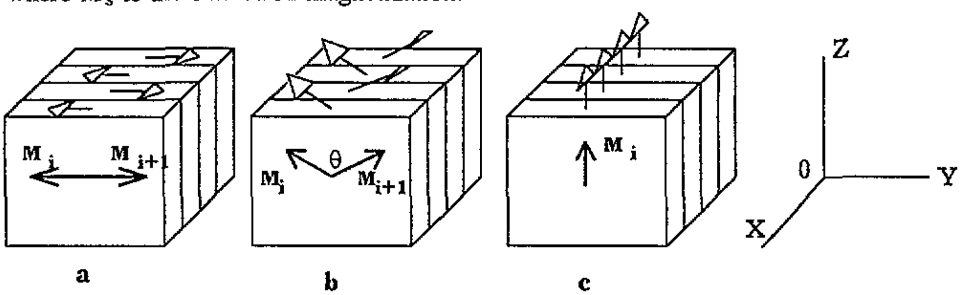


**Figure 1.** The coordinate system and the model of the superlattice used in our study. ($a$) Initially the neighbouring layers are magnetized antiparallel. ($b$) When applied magnetic field $H$ is less than the saturation field $H_s$, the neighbouring magnetizations are rotated through an angle $0 < \theta < \pi$ relative to each other. ($c$) If magnetic field is strong enough ($H \geqslant H_s$), the magnetizations are forced to lie in the parallel arrangement.

We introduce the maximum magnetoresistance ratio $\Delta^G$ as

$$\Delta^G = [\rho^G(H_s) - \rho^G(0)]/\rho^G(0) \tag{2}$$

where $H_s$ is the saturation field, and a dimensionless function $\delta^G(H)$ describing the field dependence of the magnetoresistance

$$\delta^G(H) = [\rho^G(H) - \rho^G(0)]/[\rho^G(H_s) - \rho^G(0)]. \tag{3}$$

Here G defines the geometry under discussion (CIP or CPP). Having obtained $\delta^G(H)$ and $\mu(H)$ from experimental $\rho^G(H)$ and $M(H)$ dependences, one can find a correlation between $\delta^G$ and $\mu$ and eliminate the common variable $H$. When experimental data are represented in the form $\delta(\mu^2)$, they should be compared with results of the present theory to estimate microscopic parameters of the theory, as has been demonstrated with the CIP geometry in [13].

Let $\varepsilon_\sigma(k)$ be a spectrum of conduction electrons with spin opposite to $M_i$ ($\sigma = +$) or along $M_i$ ($\sigma = -$) in the ferromagnetic metal and $A_\pm$ be corresponding Fermi surfaces defined by the equations $\varepsilon_\sigma(\boldsymbol{k}) = \zeta$ in the quasi-momentum space, $\zeta$ being the chemical potential. Transport properties of the ferromagnet depend on the electron velocity $\boldsymbol{v} = \partial\varepsilon_\sigma/\partial\boldsymbol{k}$ at the Fermi surface and the intralayer relaxation time of momentum $\tau_\sigma$.

The rigorous semiclassical treatment of the superlattice response to an applied electric field $E$ requires solving a system of equations for non-equilibrium parts of the distribution function in each layer $\phi_\sigma^{(i)}(x, \boldsymbol{k})$ together with a set of boundary conditions that establish a link between the distribution function of electrons moving away from the boundary and that of electrons incident on the boundary from both layers. Such boundary conditions have been derived in [14].

The equation for $\phi_\sigma^{(i)}$ in the relaxation-time approximation takes the form

$$v_x \partial\phi_\sigma^{(i)}/\partial x + e\boldsymbol{v}\cdot\boldsymbol{E}(x)\delta(\varepsilon_\sigma - \zeta) = -(\phi_\sigma^{(i)} - \langle\phi_\sigma^{(i)}\rangle)/\tau_\sigma \tag{4}$$

where $\langle\phi_\sigma^{(i)}\rangle$ is the local-equilibrium part from $\phi_\sigma^{(i)}$ defined by

$$\langle\phi_\sigma^{(i)}\rangle = \delta(\varepsilon_\sigma - \zeta)\int \mathrm{d}\boldsymbol{k}\,\phi_\sigma^{(i)} \Big/ \int \mathrm{d}\boldsymbol{k}\,\delta(\varepsilon_\sigma - \zeta). \tag{5}$$

The integrals in (5) are taken over the Fermi surface. In the subsequent discussion, as we are interested in formulating a realistic model of the GMR, we ignore intralayer spin-relaxation processes, and the layer thickness $L$ is assumed to be far less than the spin-diffusion length.

To write the boundary conditions, we introduce quantities characterizing the interaction of electrons with the interface. Let $R_\sigma$ ($P_\sigma$) be the specular (diffusive) reflection probability for an electron of spin $\sigma$ and $T_{\sigma_1\sigma_2}$ ($Q_{\sigma_1\sigma_2}$) be the probability for an electron of spin $\sigma_1$ (with respect to the magnetization $\boldsymbol{M}_i$) in layer $i$ to pass coherently (diffusively) through the interface into spin state $\sigma_2$ (with respect to $\boldsymbol{M}_{i+1}$) in layer $i+1$. The quantities introduced are subject to the normalization condition

$$R_\sigma + T_{\sigma\sigma} + T_{(-\sigma)\sigma} + P_\sigma + Q_{\sigma\sigma} + Q_{(-\sigma)\sigma} = 1. \tag{6}$$

The boundary condition at the interface $x = x_i$ can be written in the form:

$$\begin{aligned}\phi_\sigma^{(i)}(k_x) = {} & R_\sigma\phi_\sigma^{(i)}(-k_x) + \sum_{\sigma_1} T_{\sigma\sigma_1}\phi_{\sigma_1}^{(i-1)}(k_x) + P_\sigma\langle\phi_\sigma^{(i)}(-k_x)\rangle_{v_x<0} \\ & + \sum_{\sigma_1} Q_{\sigma\sigma_1}\langle\phi_{\sigma_1}^{(i-1)}(k_x)\rangle_{v_x>0}\end{aligned} \tag{7}$$

where $k_x$ is the $x$ component of momentum at the Fermi surface with $v_x > 0$, and brackets $\langle\ldots\rangle_{v_x>0}$ denote averaging in a matter like (5) over the part of the Fermi surface with $v_x > 0$. The boundary condition (7) provides the continuity of streams of particles through the interface [14].

Having found the various $\phi$ from (4)–(7), one can calculate the non-equilibrium charge density $n(x)$ and the current density $j(x)$ by using

$$n^{(i)}(x) = \frac{e}{(2\pi\hbar)^3}\int \mathrm{d}\boldsymbol{k}\,[\phi_+^{(i)}(x, \boldsymbol{k}) + \phi_-^{(i)}(x, \boldsymbol{k})] \tag{8}$$

$$j^{(i)}(x) = \frac{e}{(2\pi\hbar)^3}\int \mathrm{d}\boldsymbol{k}\,\boldsymbol{v}[\phi_+^{(i)}(x, \boldsymbol{k}) + \phi_-^{(i)}(x, \boldsymbol{k})]. \tag{9}$$

The calculation techniques to be used are different for different geometries. In the CIP geometry ($j \parallel OZ$) the electric field $E_0$ is uniform but there is non-uniform current density $j$. Having averaged $j(x)$ over the layer thickness

$$J = \frac{1}{L} \int_{x_i}^{x_i+L} \mathrm{d}x \, j_z^{(i)}(x)$$

one obtains the conductivity $\sigma^{\mathrm{CIP}}$ and the specific resistance $\rho^{\mathrm{CIP}}$ as

$$J = \sigma^{\mathrm{CIP}} E_0 \qquad \rho^{\mathrm{CIP}} = (\sigma^{\mathrm{CIP}})^{-1}. \tag{10}$$

As to the CPP geometry ($j \parallel OX$), we have uniform current density $j_0$ but non-uniform electric field $E(x)$. The charge density $n(x)$ given by (8) is a functional of $E(x)$. By introducing the electrostatic potential $V(x)$ through the equation $E(x) = -\mathrm{d}V(x)/\mathrm{d}x$ and using the Poisson equation $\mathrm{d}^2V(x)/\mathrm{d}x^2 = -4\pi n(x)$, we come to a differential equation for $V(x)$ to be solved within the layer $x_i \leqslant x \leqslant x_i + L$. The total distribution of $V(x)$ throughout the multilayer can be deduced by analogy. The specific resistance is defined by

$$\rho^{\mathrm{CPP}} = [V(-L/2) - V(L/2)]/Lj_0. \tag{11}$$

The fundamental difference between the CIP and CPP problems is in the existence of charge accumulation near the interfaces in the latter case. The charge screening effects give rise to sharp short-range inhomogeneities of the electric potential, field, etc. The unique damping length of the inhomogeneities is the Debye screening length $r_{\mathrm{D}}$, which should amount to several ångströms for usual metals. If $r_{\mathrm{D}}$ and $L$ are of the same order, one would expect to find unusual transport properties of the superlattice. If this is not the case and $L \gg r_{\mathrm{D}}$, the magnetoresistance is little affected by the specific behaviour of the potential near the interfaces. According to the usual approach, the potential can be treated as if it had jumps at the interfaces $x = x_i$ [12]. In this approximation the position dependence of $V(x)$ can be evaluated from the equation $n(x) = 0$ instead of the Poisson equation.

## 3. The CIP and CPP giant magnetoresistance: a resistance scheme for an arbitrary angle between the layer magnetizations

The Fermi surface having a complicated form in ferromagnets, one of the main difficulties encountered in the calculation is to take integrals over the Fermi surfaces $A_+$ and $A_-$. To obtain an analytical expression of the GMR, we have to take one or other simple model of the electron spectrum. In our approach the electron energy spectrum $\varepsilon_\sigma(\boldsymbol{k})$ is assumed to have the form corresponding to the octahedral model of the Fermi surface:

$$\varepsilon_\pm(\boldsymbol{k}) = v^\pm(|k_x| + |k_y| + |k_z|) \mp \varepsilon_{\mathrm{a}} \tag{12}$$

where $\varepsilon_{\mathrm{a}}$ is the spin-splitting energy. The key feature of the model (12) is in the independence of the Fermi velocity of electrons from $\boldsymbol{k}$. To simplify the calculation, in the following we also ignore any difference between the Fermi velocities and the Fermi-surface areas for electrons of opposite spin: $A_+ = A_- = A$, $v^+ = v^- = v_{\mathrm{F}}$. However, our calculation can be extended without essential change to the general case.

The analytical formulae for the CIP-GMR in the model (12) have been obtained in [13]. It can be shown that the resistivity $\rho^{G}$ has the same form in both the CIP and CPP geometries:

$$\rho^{G} = \frac{\rho_{+}^{G}\rho_{-}^{G} + (\rho_{+}^{G} + \rho_{-}^{G})\rho_{\rm mix}^{G}}{\rho_{+}^{G} + \rho_{-}^{G} + 4\rho_{\rm mix}^{G}}. \tag{13}$$

Here the 'effective resistivity' $\rho_{\pm}^{G}$ is given by the sum

$$\rho_{\sigma}^{G} = \rho_{\sigma} + r_{\sigma}^{G} \tag{14}$$

where $\rho_{\sigma} = (2\pi\hbar)^{3}/e^{2}Av_{F}\tau_{\sigma}$ is the resistivity of the spin-subzone $\sigma$ in the bulk ferromagnet and $r_{\sigma}^{G}$ is an interface contribution. The resistivity $\rho_{\rm mix}^{G}$ has been introduced to take into account 'mixing' processes, which are due to the transmission of electrons between layers with different $M$. The formula of equation (13) corresponds to the resistance scheme of figure 2.

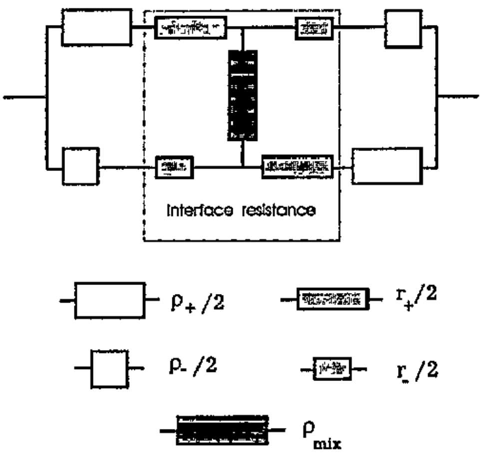


**Figure 2.** Resistor network analogy for the magnetoresistance of equation (13).

Being considered in different geometries, the interface contributions to $\rho_{\sigma}^{G}$ and $\rho_{\rm mix}^{G}$ are determined by different combinations of probabilities $P$, $R$, $Q$ and $T$. In the case of the CPP geometry the total penetration probabilities $W_{\sigma} = T_{\sigma\sigma} + Q_{\sigma\sigma}$ and $W = T_{+-} + Q_{+-}$ that describe the electron transmission from spin-subzone $\sigma$ in one layer to spin-subzone $\sigma$ and $(-\sigma)$ respectively in the neighbouring layer are of importance:

$$r_{\sigma}^{\rm CPP} = \frac{W_{-\sigma} - [W_{\sigma}W_{-\sigma} + \frac{1}{2}W(W_{\sigma} + W_{-\sigma})]}{W_{\sigma}W_{-\sigma} + \frac{1}{2}W(W_{\sigma} + W_{-\sigma})}2r_{L} \tag{15}$$

$$\rho_{\rm mix}^{\rm CPP} = \frac{W}{W_{+}W_{-} + \frac{1}{2}W(W_{+} + W_{-})}r_{L} \tag{16}$$

where $r_{L} = (2\pi\hbar)^{3}/e^{2}AL$. With the CIP problem, only the total probability of the diffusive scattering $S_{\sigma} = P_{\sigma} + Q_{\sigma\sigma} + Q_{(-\sigma)\sigma}$ and the factor $T_{+-}$ are crucial:

$$r_{\sigma}^{\rm CIP} = S_{\sigma}r_{L} \tag{17}$$

$$\rho_{\rm mix}^{\rm CIP} = T_{+-}r_{L}. \tag{18}$$

## 4. The CIP and CPP magnetoresistance: the angular dependence

To obtain the magnetoresistance versus magnetization dependence, we have to specify the angular dependence of the probabilities in equations (15)–(18). To this end, we identify first the angular dependence that is due to rotating the spin quantization direction of an electron in travelling from one layer to its neighbour. It is easy to show [13] that, being equal to 1 *in the initial system of coordinates, the probability* $\eta$ *of finding an electron with spin along* the magnetization $M$ in a new system rotated through an angle $\theta$ is equal to $\eta = \cos^2(\theta/2)$.

Generally, the penetration probabilities can be represented in the form:

$$\begin{aligned} T_{\sigma\sigma}(\theta) &= T_\sigma(\theta)\cos^2(\theta/2) \qquad & T_{\sigma(-\sigma)}(\theta) &= T_\sigma(\theta)\sin^2(\theta/2) \\ Q_{\sigma\sigma}(\theta) &= Q_\sigma(\theta)\cos^2(\theta/2) \qquad & Q_{\sigma(-\sigma)}(\theta) &= Q_\sigma(\theta)\sin^2(\theta/2). \end{aligned} \tag{19}$$

The phenomenological parameters $T_\sigma$ and $Q_\sigma$ characterize properties of the interlayer, interfacial roughness, band structure, etc. Generally speaking, they are functions of $\theta$. To find the specific dependences of $T_\sigma(\theta)$ and $Q_\sigma(\theta)$, one should consider a quantum-mechanical model of interface scattering. Here we shall restrict our consideration to the simplest approximation.

By taking into account that $T_{\sigma\sigma}$ and $Q_{\sigma\sigma}$ are of interest to us in the field of small $\theta$, we expand each of $T_\sigma$ and $Q_\sigma$ as a power series in $\theta$ and restrict ourselves to the first non-vanishing term:

$$T_\sigma(\theta) \simeq t_\sigma^{\mathrm{F}} \qquad Q_\sigma(\theta) \simeq q_\sigma^{\mathrm{F}} \qquad \text{small } \theta. \tag{20}$$

On the other hand, $T_{\sigma(-\sigma)}$ and $Q_{\sigma(-\sigma)}$ are of interest for $\theta$ near $\pi$. By expanding $T_\sigma$ and $Q_\sigma$ in powers of $(\theta - \pi)$, we obtain

$$T_\sigma(\theta) \simeq t^{\mathrm{AF}} \qquad Q_\sigma(\theta) \simeq q^{\mathrm{AF}} \qquad \theta \simeq \pi. \tag{21}$$

By using asymptotics of the kind (20) and (21) over the whole range of $\theta$, we find the interpolation formulae:

$$\begin{aligned} T_{\sigma\sigma}(\theta) &= t_\sigma^{\mathrm{F}}\cos^2(\theta/2) \qquad & T_{\sigma(-\sigma)} &= t^{\mathrm{AF}}\sin^2(\theta/2) \\ Q_{\sigma\sigma}(\theta) &= q_\sigma^{\mathrm{F}}\cos^2(\theta/2) \qquad & Q_{\sigma(-\sigma)} &= q^{\mathrm{AF}}\sin^2(\theta/2) \\ P_\sigma(\theta) &= p_\sigma^{\mathrm{F}}\cos^2(\theta/2) + p_\sigma^{\mathrm{AF}}\sin^2(\theta/2). \end{aligned} \tag{22}$$

Here the various $t$, $q$ and $p$ are parameters of the theory.

The difference between the interface contributions to the 'effective resistivity' $\rho_\sigma^{\mathrm{G}}$ in different geometries cannot result in any qualitative difference in the behaviour of the CIP and CPP magnetoresistance. In contrast, the fundamental difference in their behaviour results from the fact that the $\theta$ dependence of $\rho_{\mathrm{mix}}^{\mathrm{CPP}}$ differs essentially from that of $\rho_{\mathrm{mix}}^{\mathrm{CIP}}$.

The resistivity $\rho_{\mathrm{mix}}^{\mathrm{CIP}}(\theta)$ is a limited function and peaks at $\theta = \pi$. In the approximation (22):

$$\rho_{\mathrm{mix}}^{\mathrm{CIP}}(\theta) = t^{\mathrm{AF}} r_{\mathrm{L}} \sin^2(\theta/2). \tag{23}$$

To the contrary, $\rho_{\mathrm{mix}}^{\mathrm{CPP}}(\theta)$ increases without limit at $\theta \to \pi$. In the approximation (22) it takes the form

$$\rho_{\mathrm{mix}}^{\mathrm{CPP}} = \frac{r_L \tan^2(\theta/2)}{(w_+^{\mathrm{F}} w_-^{\mathrm{F}}/w^{\mathrm{AF}})\cos^2(\theta/2) + \frac{1}{2}(w_+^{\mathrm{F}} + w_-^{\mathrm{F}})\sin^2(\theta/2)} \tag{24}$$

where we have introduced $w_\sigma^{\mathrm{F}} = t_\sigma^{\mathrm{F}} + q_\sigma^{\mathrm{F}}$, $w^{\mathrm{AF}} = t^{\mathrm{AF}} + q^{\mathrm{AF}}$. By way of illustration of the main physical features of the foregoing results, let us take a look at several important limiting cases.

*4.1. Superlattices with very highly permeable perfect interfaces*

The simplest case of a multilayer with very highly permeable perfect interfaces will be our initial concern. Both the diffusive and specular interface scattering are suggested to be so insignificant that electrons move coherently through the interfaces without essentially any resistance, so the following condition should be met:

$$S_\sigma, 1 - W_\sigma, 1 - W \ll \min(1, L/l_\sigma) \tag{25}$$

where $l_\sigma = v_F \tau_\sigma$ is the electron mean free path. In equations (13)–(18) we come to the approximation:

$$r_\sigma^G \simeq 0 \qquad \rho_{\rm mix}^{\rm CIP} \simeq r_L \sin^2(\theta/2) \qquad \rho_{\rm mix}^{\rm CPP} \simeq r_L \tan^2(\theta/2). \tag{26}$$

Substitution of equation (26) into equation (13) yields the following expressions for the resistivity:

$$\begin{aligned} \rho^{\rm CIP} &= \frac{\rho_+\rho_- + (\rho_+ + \rho_-) r_L \sin^2(\theta/2)}{\rho_+ + \rho_- + 4 r_L \sin^2(\theta/2)} \\ \rho^{\rm CIP} &= \frac{\rho_+\rho_- + (\rho_+ + \rho_-) r_L \tan^2(\theta/2)}{\rho_+ + \rho_- + 4 r_L \tan^2(\theta/2)}. \end{aligned} \tag{27}$$

The magnetoresistance is only due to the difference between spin-majority and spin-minority electrons in their intralayer transport properties, being specified by two parameters $\lambda_\sigma = l_\sigma / L \equiv r_L/\rho_\sigma$. By using the definition of the GMR magnitude

$$\Delta^G = [\rho^G(\theta = 0) - \rho^G(\theta = \pi)]/\rho^G(\theta = \pi)$$

we obtain

$$\Delta^{\rm CIP} = -\left(\frac{\rho_+ - \rho_-}{\rho_+ + \rho_-}\right)^2 \frac{1}{1 + (\lambda_+ + \lambda_-)^{-1}} \qquad \Delta^{\rm CPP} = -\left(\frac{\rho_+ - \rho_-}{\rho_+ + \rho_-}\right)^2. \tag{28}$$

We recall that equations (28) hold good at $L \ll l_{\rm sf}$ (the spin-diffusive length); if this is not the case, $\Delta^{\rm CPP}$ vanishes, as shown in [12]. There are fundamental differences between the CIP-GMR and CPP-GMR in their thickness dependence: the effect vanishes at $L \sim l_\sigma$ in the former case but at $L$ of the order of $l_{\rm sf}$ in the latter. With $l_{\rm sf} \gg l_\sigma$ always being the case, we find $\Delta^{\rm CIP}/\Delta^{\rm CPP} \simeq (\lambda_+ + \lambda_-) \ll 1$ at $l_\sigma \ll L \ll l_{\rm sf}$.

The magnetoresistance versus magnetization dependence is defined by the function

$$\delta^G(\mu) = \frac{\alpha^G \mu^2}{1 - (1 - \alpha^G)\mu^2} \tag{29}$$

where

$$\alpha^{\rm CPP} = \frac{\lambda_+ + \lambda_-}{4\lambda_+\lambda_-} \qquad \alpha^{\rm CIP} = \frac{\lambda_+ + \lambda_-}{4\lambda_+\lambda_- + \lambda_+ + \lambda_-} = \frac{\alpha^{\rm CPP}}{1 + \alpha^{\rm CPP}}. \tag{30}$$

As may be seen from (29) and (30), the condition $\delta^{\rm CIP}(\mu) < \delta^{\rm CPP}(\mu)$ is always met. In the thin-layers limit $\lambda_\sigma \gg 1$ both curves $\delta^G(\mu)$ have a sharp increase within a narrow range near $\mu = 1$; that is, the magnetoresistance rises sharply as the angle $\theta$ between the neighbouring magnetizations is varied through a small range $\Delta\theta$ near $\theta = \pi$. The value $\Delta\theta$ can be estimated as $\Delta\theta \simeq [(\lambda_+ + \lambda_-)/\lambda_+\lambda_-]^{1/2} \ll \pi$, and it may be deduced that the GMR might occur, even though the initial magnetic arrangement (at $H = 0$) is far from antiparallel.

Differing little from the curve $\delta^{\mathrm{CPP}}(\mu)$ at $\lambda_\sigma \gg 1$, the curve $\delta^{\mathrm{CIP}}(\mu)$ lies under the curve $\delta = \mu^2$, tending to it in the limit $\lambda_\sigma \ll 1$. In contrast, if $\lambda_\sigma \ll 1$ is fulfilled, the function $\delta^{\mathrm{CPP}}(\mu)$ rises steeply from 0 to 1 near $\mu = 0$ with a characteristic length $\mu_0$, which, being defined by the equation $\delta^{\mathrm{CPP}}(\mu_0) = 1/2$, is equal to $\mu_0 = 2[\lambda_+\lambda_-/(\lambda_+ + \lambda_-)]^{1/2}$. By this means the magnetoresistance may peak at a field $H_0$ less than the saturation field $H_s$. If it is granted that in fields $H \ll H_s$ the relative magnetization is linear in the magnetic field $\mu(H) \propto H/H_s$, the fresh damping scale is defined as

$$H_0 = 2H_s[\lambda_+\lambda_-/(\lambda_+ + \lambda_-)]^{1/2}. \qquad (31)$$

The variations of the curves $\delta^{\mathrm{G}}(\mu)$ with factor $\lambda_+$ appear in figure 3.

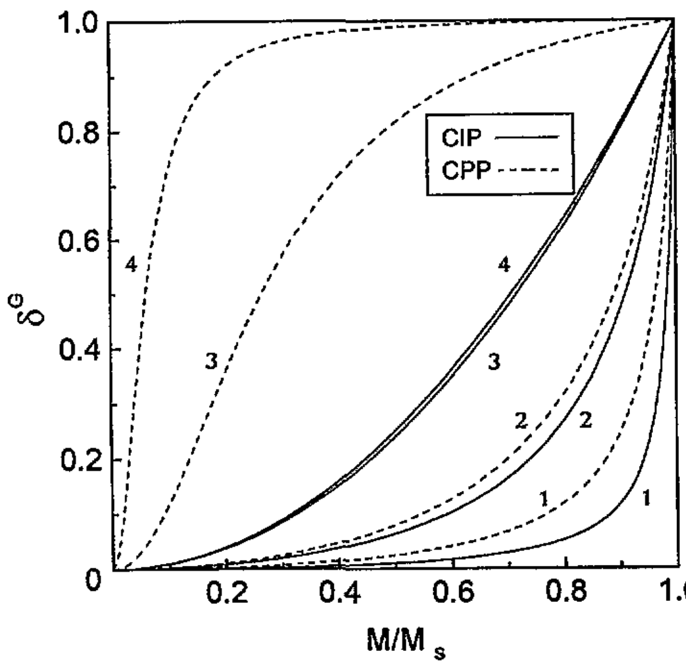


**Figure 3.** Normalized magnetoresistance $\delta$ as a function of the relative magnetization $M/M_s$ for different geometries in the case of perfect interfaces. The full and broken curves correspond respectively to the CIP and CPP geometry. The curves have been calculated for $t^{\mathrm{AF}} = t^{\mathrm{F}}_{\pm} = 0.9$, $q^{\mathrm{AF}} = q^{\mathrm{F}}_{\pm} = p^{\mathrm{F}}_{\pm} = p^{\mathrm{AF}}_{\pm} = 0$, $l_-/l_+ = 6$. The ratio $L/l_+$ has been taken as (1) 0.1, (2) 0.8, (3) 50 and (4) 1000.

## 4.2. *Superlattices with only slightly permeable interfaces*

Let us take up the case of strong interface scattering

$$W_\sigma, W \ll \min\{1, l_\sigma/L\} \qquad (32)$$

in which the difference between the CIP and CPP problems is clearly defined. As to the CPP geometry, diffusive interface scattering is equal to specular scattering, the interface resistance being determined by the total probabilities of electron transmission $W_\sigma$ and $W$. Under the condition (32) the CPP-GMR always has its origin in the interface scattering and its behaviour is determined completely by three parameters: $w^{AF}$, $\eta = (w^F_+ + w^F_-)/2w^{AF}$ and $\xi = w^F_+ w^F_-/(w^{AF})^2$. By neglecting $\rho_\sigma$ in equation (14), one finds

$$\Delta^{CPP} = -\left(1 - \frac{1 - 2\eta w^{AF} + \xi (w^{AF})^2}{(1 - w^{AF})(\eta - \xi w^{AF})}\right) \tag{33}$$

the magnetization dependence having the form

$$\delta^{CPP}(\mu) = \frac{\alpha^{CPP}\mu^2 + \beta^{CPP}\mu^4}{1 - (1 - \alpha^{CPP} - \beta^{CPP} - \gamma^{CPP})\mu^2 - \gamma^{CPP}\mu^4} \tag{34}$$

where

$$\alpha^{CPP} = \frac{(\eta - 1)(\eta - \xi w^{AF})}{\eta - 1 + w^{AF}(\eta - \xi)} \tag{35}$$

$$\beta^{CPP} = \frac{w^{AF}(\eta - \xi)(\eta - \xi w^{AF})}{\eta - 1 + w^{AF}(\eta - \xi)} \tag{36}$$

$$\gamma^{CPP} = -w^{AF}(\eta - \xi). \tag{37}$$

Having found $\alpha^{CPP}$, $\beta^{CPP}$, $\gamma^{CPP}$ and $\Delta^{CPP}$ by an experiment, one can immediately identify the microscopic parameters $w^{AF}$ and $w^F_\pm$ by the following formulae:

$$w^{AF} = 1 - \gamma^{CPP}/(\Delta^{CPP}\beta^{CPP}) \tag{38}$$

$$w^F_\pm = w^{AF}[\eta \pm (\eta^2 - \xi)^{1/2}] \tag{39}$$

where

$$\eta = 1 - \frac{\alpha^{CPP}\gamma^{CPP}}{2\beta^{CPP} - \gamma^{CPP}/\Delta^{CPP}} \tag{40}$$

$$\xi = \eta + \gamma^{CPP}/w^{AF}. \tag{41}$$

With the CIP geometry, of vital importance is what kind of interface scattering prevails. If the scattering is specular, coherent reflection makes no contribution to the resistance ($r^{CIP}_\sigma \simeq 0$), so that $\rho^{CIP}$ originates only from the intralayer scattering. However, the probability of an electron passing through layers having different $M$ decreases, resulting in a reduction in the GMR compared with (28):

$$\Delta^{CIP} = -\left(\frac{\rho_+ - \rho_-}{\rho_+ + \rho_-}\right)^2 \frac{t^{AF}}{t^{AF} + (\lambda_+ + \lambda_-)^{-1}}. \tag{42}$$

The function $\delta^{CIP}(\mu)$ is given by formula (29) again, but we have

$$\alpha^{CIP} = (\lambda_+ + \lambda_-)/(\lambda_+ + \lambda_- + 4t^{AF}\lambda_+\lambda_-). \tag{43}$$

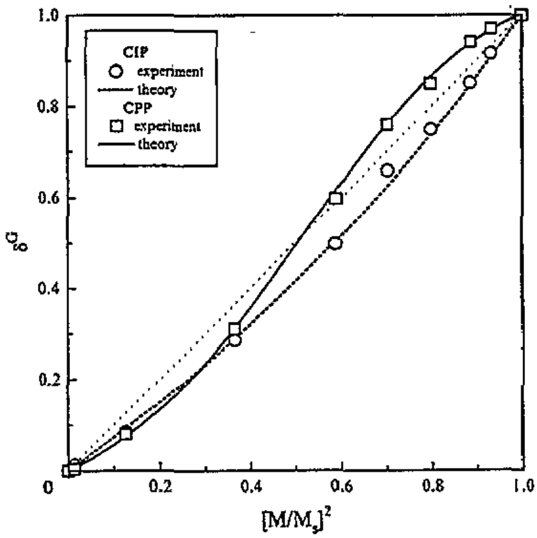


**Figure 4.** Theoretical (curves) and experimental (points) normalized magnetoresistance in Ag/Co [3] as a function of $(M/M_s)^2$. The experimental data for the CIP magnetoresistance (O) fits the theoretical curve (- - -) of kind (45) with the parameters $\alpha^{\mathrm{CIP}} = 0.76$, $\beta^{\mathrm{CIP}} = 0$; the experimental results for the CPP magnetoresistance (□) correspond to the theoretical curve (——) of kind (34) with the parameters $\alpha^{\mathrm{CPP}} = 0.46$, $\beta^{\mathrm{CPP}} = 0.39$, $\gamma^{\mathrm{CPP}} = -1.04$.

Alternatively, if diffusive scattering prevails ($S_\sigma \gg L/l_\sigma$), the behaviour of the magnetoresistance is determined by the total probability of interface diffusive scattering $s_\sigma = p_\sigma + q_\sigma$ and we have

$$\Delta^{\mathrm{CIP}} = \frac{a_+ + aa_- + (1+a)a_+a_-}{1 + a_- + a(1+a_+)} \tag{44}$$

where we have introduced $a_\sigma = (s_\sigma^{\mathrm{F}} - s_\sigma^{\mathrm{AF}})/s_\sigma^{\mathrm{AF}}$ and $a = s_+^{\mathrm{AF}}/s_-^{\mathrm{AF}}$. The magnetoresistance versus magnetization dependence takes the form

$$\delta^{\mathrm{CIP}}(\mu) = \frac{\alpha^{\mathrm{CIP}}\mu^2 + \beta^{\mathrm{CIP}}\mu^4}{1 - (1 - \alpha^{\mathrm{CIP}} - \beta^{\mathrm{CIP}})\mu^2} \tag{45}$$

where

$$\alpha^{\mathrm{CIP}} = (a_+ + aa_-)/[(1+a)\Delta^{\mathrm{CIP}}] \qquad \beta^{\mathrm{CIP}} = a_+a_-/\Delta^{\mathrm{CIP}}. \tag{46}$$

### 4.3. *Superlattices with arbitrary interfaces*

Generally, whatever the proportion of transmission and scattering at the interfaces may be, it can be shown that $\delta^{\mathrm{CPP}}(\mu)$ is always of the form (34) but factors $\alpha^{\mathrm{CPP}}$, $\beta^{\mathrm{CPP}}$, $\gamma^{\mathrm{CPP}}$ as well as the GMR magnitude $\Delta^{\mathrm{CPP}}$ depend on five microscopic parameters ($w^{\mathrm{AF}}$, $w_\pm^{\mathrm{F}}$, $\lambda_\pm$) in a complicated way. In the general case, $\delta^{\mathrm{CIP}}(\mu)$ takes the form (45), where $\alpha^{\mathrm{CIP}}$, $\beta^{\mathrm{CIP}}$ as well as $\Delta^{\mathrm{CIP}}$ depend on seven microscopic parameters ($t^{\mathrm{AF}}$, $s_\pm^{\mathrm{AF}}$, $s_\pm^{\mathrm{F}}$, $\lambda_\pm$). By parametrizing experimental curves in accordance with (34) and (45), one can try to reduce several combinations of microscopic parameters from $\Delta^{\mathrm{G}}$, $\alpha^{\mathrm{G}}$, $\beta^{\mathrm{G}}$ and $\gamma^{\mathrm{G}}$. An example of such a procedure with the CIP geometry has been given in [13].

We take as our example the experimental data of Pratt *et al* [3] who observe $\Delta^{\rm CIP} = -0.127$ and $\Delta^{\rm CPP} = -0.415$ in Ag/Co. The experimental data for the CIP and CPP magnetoresistance [3] and corresponding theoretical curves are depicted in figure 4. The fitting parameters have been found to be $\alpha^{\rm CPP} = 0.46$, $\beta^{\rm CPP} = 0.39$, $\gamma^{\rm CPP} = -1.04$, $\alpha^{\rm CIP} = 0.76$ and $\beta^{\rm CIP} = 0$. In fitting we used the experimental values of the derivative $\mathrm{d}\delta/\mathrm{d}(\mu^2)$ at $\mu = 0$ and $\mu = 1$ and (for the CPP case) the intersection point of the curves $\delta^{\rm G}(\mu)$ and $\delta_0 = \mu^2$ whereby the set of parameters $\alpha$, $\beta$, $\gamma$ is uniquely determined. Having analysed the results of the fit, we can give some conclusions about the nature of the GMR effect in Ag/Co [3]. It can be shown in a general way that the value $\beta^{\rm CIP}$ is defined by the asymmetry in diffusive scattering. The fact that $\beta^{\rm CIP} = 0$ allows us to say with certainty that there is no asymmetry in diffusive scattering, that is, diffusive scattering is spin-independent and does not depend on what kind of magnetic ordering appears in the superlattice ($s^{\rm F}_{\pm} = s^{\rm AF}_{\pm} = s$). So the CIP-GMR originates from bulk spin-dependent scattering. By assuming that diffusive scattering is insignificant ($s \ll L/l_\sigma$) and the equations (42) and (43) remain valid, we obtain the following estimates of microscopic parameters: $l_-/l_+ = 4.2$, $t^{\rm AF}L/l_- = 0.09$, $t^{\rm AF}L/l_+ = 0.41$. With the CPP-GMR, it is significant that both interface (from specular reflection) and intralayer contributions to the resistivity play important parts in the magnetoresistance behaviour. Unfortunately, there are many unknown parameters in the CPP case, and it seems to be difficult to estimate them from the fitting parameters.

## 5. The thickness dependence of the magnetoresistance

The behaviour of $\Delta^{\rm CIP}$ as a function of the magnetic layer thickness $L$ essentially depends on the ratio of the $s_\sigma$ and $t^{\rm AF}$ parameters characterizing diffusive scattering and coherent penetration respectively. In the limit $s_\sigma \ll t^{\rm AF}$ the main contribution to $\Delta^{\rm CIP}$ is made by either interface scattering (if $L/l_\sigma \ll s_\sigma$) or intralayer scattering (if $L/l_\sigma \gg t^{\rm AF}$), the asymptotic behaviour of $\Delta^{\rm CIP}$ being given by

$$\Delta^{\rm CIP} \simeq -\left(\frac{\rho_+ - \rho_-}{\rho_+ + \rho_-}\right)^2 \qquad s_\sigma \ll L/l_\sigma \ll t^{\rm AF} \tag{47}$$

$$\Delta^{\rm CIP} \simeq -\left(\frac{\rho_+ - \rho_-}{\rho_+ + \rho_-}\right)^2 t^{\rm AF}\frac{l_+ + l_-}{L} \qquad L/l_\sigma \gg t^{\rm AF}. \tag{48}$$

There are two different damping lengths ($L_1 \simeq t^{\rm AF}l_\sigma$ and $L_2 \simeq l_\sigma$) in this case in the manner indicated in figure 5. If the diffusive scattering is strong and $t^{\rm AF} \ll s_\sigma$, the curve $\Delta^{\rm CIP}(L)$ decreases monotonically as illustrated in figure 6.

As for the CPP case, there is only the damping length $L^{\rm CPP} \simeq [(1 - w_\sigma)/w_\sigma]l_\sigma$. At $L \ll L^{\rm CPP}$ the interface contribution has a dominant role, and in the limit $L \gg L^{\rm CPP}$ the intralayer one plays a leading part in the magnetoresistance. So we find the asymptotes:

$$\Delta^{\rm CPP} \simeq -\left(1 - \frac{2w^{\rm AF}}{w^{\rm F}_+ + w^{\rm F}_-}\right) \qquad L \ll L^{\rm CPP} \tag{49}$$

$$\Delta^{\rm CPP} \simeq -\left(\frac{\rho_+ - \rho_-}{\rho_+ + \rho_-}\right)^2 \qquad L \gg L^{\rm CPP}. \tag{50}$$

And $\Delta^{\rm CPP}(L)$ is a monotonic function as shown in figures 5 and 6.

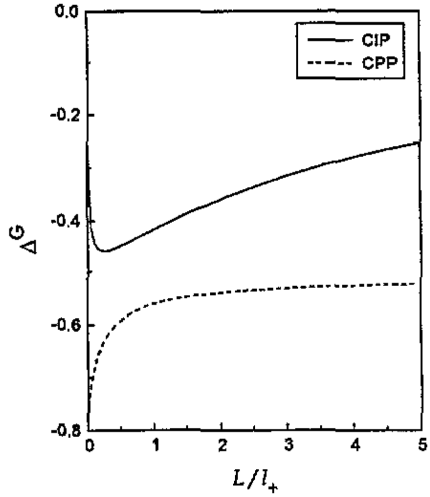


**Figure 5.** The CIP (——) and CPP (– – –) magnetoresistance ratio $\Delta^G$ as a function of $L/l_+$ in the case of perfect interfaces. The curves have been calculated for $l_-/l_+ = 6$, $q_+^F = p_+^F = p_+^{AF} = 0.01$, $q_-^F = p_-^F = p_-^{AF} = 0.003$, $q^{AF} = 0.007$, $t_+^F = t_-^F = 0.9$, $t^{AF} = 0.7$.

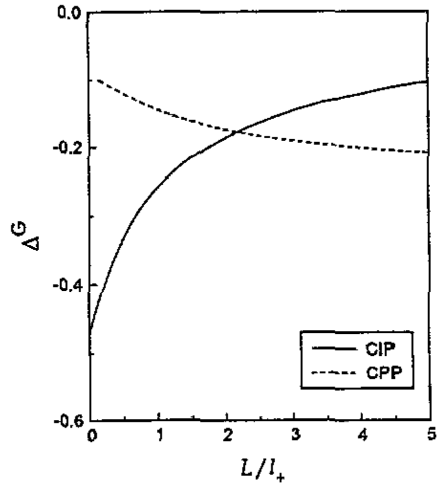


**Figure 6.** The CIP (——) and CPP (– – –) magnetoresistance ratio $\Delta^G$ as a function of $L/l_+$ in the case of strong interface scattering. The parameters assumed in our calculation are $l_-/l_+ = 3$, $q^{AF} = 0.24$, $q_+^F = p_+^F = p_+^{AF} = 0.4$, $q_-^F = p_-^F = p_-^{AF} = 0.07$, $t_+^F = 0.2$, $t_-^F = 0.6$, $t^{AF} = 0.38$.

## 6. Summary

Summing up, we have worked out a semiclassical theory in which the CIP and CPP giant magnetoresistances are studied within a unified microscopic model. The correlation between the magnetoresistance and the magnetization has been analysed and formulae that describe it in each geometry have been derived. We found the results obtained to be in good agreement with the experimental data of [3]. The behaviour of the magnetoresistance ratio as a function of the magnetic layer thickness $L$ has also been considered. The main features of our results may be summed up as follows.

The GMR magnitude in the CIP case and the same in the CPP case are defined by different sets of microscopic parameters characterizing the interface properties. Consequently, there is no definite relationship between the CIP-GMR and the CPP-GMR. As a rule, $\Delta^{CPP}/\Delta^{CIP} > 1$ but it is not inconceivable that one can discover layered systems where the CIP-GMR exceeds the CPP-GMR. The system in which this can take place should possess rough interfaces having strong spin-dependent diffusive scattering.

In superlattices with thin layers ($\lambda_\sigma \gg 1$) the GMR may be observed, even if the magnetizations are not ordered antiferromagnetically in zero magnetic field. For this to happen, the angle $\theta$ between the neighbouring magnetizations at $H = 0$ must exceed a critical value $\Delta\theta$, which can be estimated as $\Delta\theta \simeq \max\{L/l_\sigma, S_\sigma\}$ for the CIP geometry and $\Delta\theta \simeq \max\{L/l_\sigma, 1 - W_{\sigma\sigma_1}\}$ for the CPP one.

With the proviso that $L \gg l_\sigma$, it is possible to expect that the CPP magnetoresistance peaks at a magnetic field $H_0$ less than the saturation field $H_s$; crude estimates give $H_0/H_s \propto (l_\sigma/L)^{1/2} \ll 1$. As yet, there have been no experimental data on the CPP-GMR behaviour at layer thicknesses $L \geqslant l_\sigma$ and we consider it desirable to conduct such experimental investigations.

The magnetoresistance versus magnetization dependence in the CIP geometry differs essentially from that in the CPP geometry as may be inferred from (34) and (45). The curve $\delta^{CIP}(\mu)$ lies under the curve $\delta = \mu^2$, while there are no such limitations in the CPP case.

There are essential differences between $\Delta^{CIP}$ and $\Delta^{CPP}$ in their thickness dependence. The first has either one or two damping lengths, depending on the ratio of diffusive scattering to coherent transmission at the interfaces. In the limit $L \gg l_\sigma$ we always find $\Delta^{CIP} \propto L^{-1}$. Generally, there are three scaling lengths in the latter case. These are the Debye screening length, the spin diffusion length and a characteristic length $L^{CPP} \simeq [(1 - w_\sigma)/w_\sigma]l_\sigma$ at which the interface contribution to the resistance vanishes. In the limit $l_\sigma \ll L \ll l_{sf}$ one finds $\Delta^{CPP}(L) \to$ const.

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